Branching Ratios of the K^+ Meson*

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Stopping K^+ mesons were observed in a 12-in.-diam, 10-in.-deep xenon bubble chamber. The branching ratios of the $K_{\mu2}$, $K_{\mu3}$, K_{e3}, $K_{\pi2}$, τ, and τ' decay modes were calculated, giving, in percent, 63.0±0.8, 3.0±0.5, 4.7±0.3, 22.4±0.8, 5.1±0.2, and 1.8±0.2, respectively. These ratios are compared both with previous experiments and with some theoretical predictions. The ratio of the three-body pionic decay modes is consistent with the $\Delta T=\frac{1}{2}$ rule; the ratio of the three-body leptonic-decay modes is consistent with $f_-/f_+=0$; and the $K_{\mu2}$ branching ratio does not contradict a recent theory of universality in the two-body leptonic decays of the π and K mesons.

I. INTRODUCTION

THE six main decay modes of the K^+ meson are shown in the first column of Table I. The purpose of this paper is to present the results of a measurement of the branching ratios among these six decay modes. No attempt has been made to determine the branching ratios of the several rare-decay modes since many of them would be very difficult to separate from the main-decay modes. The contamination of the decay sample used in the current experiment due to the presence of rare-decay modes is negligible because the branching ratios of the rare-decay modes are of the order of a few thousandths of a percent.

The remaining columns of Table I give the results of several previous determinations of the K^+ branching ratios among the indicated decay modes, along with the results of the current experiment. The results of Birge *et al.*,[1] Alexander *et al.*,[2] and Taylor *et al.*[3] are among the most precise emulsion measurements. The experiment of Roe *et al.*,[4] hereafter called A, made use of the same bubble chamber that was used in the current experiment. However, since the current experiment was done on a different sample of film with a significantly different method of analysis, it is felt that the two experiments should be regarded as independent. One of the motivations behind the current experiment was the fact that some of the previous experimental values of the branching ratios are in disagreement by an amount significantly more than the errors assigned.

Table I shows that the present experimental results support fairly well the results of A with the exception of the $K_{\pi2}$ and $K_{\mu3}$ branching ratios, in which cases the present experiment is more in agreement with the emulsion results. However, the sums of the $K_{\pi2}$ and $K_{\mu3}$ branching ratios for the present experiment and A are in fairly good agreement. Letting $B(K_i)$ represent the branching ratio of the decay mode K_i, the present experiment gives

$$B(K_{\pi2})+B(K_{\mu3})=0.254\pm0.007\,,$$

while for A,

$$B(K_{\pi2})+B(K_{\mu3})=0.234\pm0.011\,.$$

Therefore the only major disagreement between the present experiment and A is in the $K_{\pi2}/K_{\mu3}$ branching ratio. For the present experiment,

$$B(K_{\pi2})/B(K_{\mu3})=7.6\pm1.3\,,$$

while for A,

$$B(K_{\pi2})/B(K_{\mu3})=3.9\pm0.5\,.$$

It is believed that the disagreement is due to a system-

TABLE I. Previous determinations of the K^+ branching ratios (percent).

Decay mode	Birge *et al.*[a]	Alexander *et al.*[b]	Taylor *et al.*[c]	Roe *et al.*[d]	Present experiment
$K_{\mu2}\to\mu^++\nu$	58.5 ±3.0	56.9 ±2.6		64.2 ±1.3	63.0 ±0.8
$K_{\pi2}\to\pi^++\pi^0$	27.7 ±2.7	23.2 ±2.2		18.6 ±0.9	22.4 ±0.8
$K_{\mu3}\to\mu^++\pi^0+\nu$	2.8 ±1.0	5.9 ±1.3	2.8 ±0.4	4.8 ±0.6	3.0 ±0.5
$K_{e3}\to e^++\pi^0+\nu$	3.2 ±1.3	5.1 ±1.3		5.0 ±0.5	4.7 ±0.3
$\tau^+\to2\pi^++\pi^-$	5.6 ±0.4	6.8 ±0.4	5.2 ±0.3	5.7 ±0.3	5.1 ±0.2
$\tau^{+\prime}\to\pi^++2\pi^0$	2.1 ±0.5	2.2 ±0.4	1.5 ±0.2	1.7 ±0.2	1.8 ±0.2
$K_{\mu3}/K_{e3}$	0.88 ±0.47	1.16 ±0.39		0.96 ±0.15	0.63 ±0.10
$\tau^{+\prime}/\tau^+$	0.375±0.093	0.324±0.062	0.288±0.042	0.298±0.038	0.350±0.039

[a] See Ref. 1. [b] See Ref. 2. [c] See Ref. 3. [d] See Ref. 4.

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[1] R. W. Birge, D. H. Perkins, J. E. Peterson, D. H. Stork, and M. W. Whitehead, Nuovo Cimento 4, 834 (1959).
[2] G. Alexander, R. H. W. Johnson, and C. O'Ceallaigh, Nuovo Cimento 6, 478 (1957).
[3] S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).
[4] B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961).

atic error in A in the calculation of $P(K_{\pi2})$, the probability that a $K_{\pi2}$ will pass the $K_{\pi2}$ kinematical test, which will be discussed later. In the present experiment, $P(K_{\pi2})$ was calculated by simulating measured $K_{\pi2}$ decays by means of Monte Carlo calculations. (We call these simulated decays "Monte Carlo decays.")

Table I shows that the present experiment does not agree with the emulsion experiments in the $K_{\mu2}$ branching ratio.

In the τ'/τ branching ratio, all of the indicated experiments are consistent with the $\Delta T=\frac{1}{2}$ rule which predicts a value of 0.325. The implications of the $K_{\mu3}/K_{e3}$ branching ratio obtained in the present experiment will be discussed in Sec. VI.

The results of the present experiment are based on the analysis of a sample of 21 000 K^+-meson decays obtained from the exposure of a 12-in.-diam, 10-in.-deep xenon bubble chamber to a separated beam of K^+ mesons of momentum 700 MeV/c at the Berkeley Bevatron in 1959. A moderator was used to reduce the entrance-beam momentum to about 400 MeV/c, so that the K^+ mesons stopped and decayed near the center of the chamber. The chamber had no magnetic field. The high efficiency for conversion of gamma rays from π^0 mesons into electron pairs and the ease of recognition of electrons in xenon have been used to separate the various K^+ decay modes in this experiment.[5] About 10 600 four-view stereophotographs were used.

II. THE INITIAL K^+-DECAY SAMPLE

This section describes how the initial decay sample containing about 21 000 K^+ decays was obtained.

Initial Scan

Each picture was scanned independently by two professional scanners. Two selective scanning rules were used on this initial scan in order to reduce some scanning biases. Firstly, only those decays for which the kaon entered the chamber with a deviation from the beam direction of less than 20° on all four views as measured on the scanning projector were counted. This helped to reduce the π^+ contamination because a π^+ is more likely than a K^+ to scatter through large angles before entering the chamber and also because those π^+'s coming from K^+'s decaying in flight before entering the chamber will deviate from the beam direction. Secondly, pictures having seven or more countable K^+ decays on them were discarded in order to reduce gamma-ray and secondary ambiguities.

π^0's from K^+ decays were detected by the electron pairs formed by the gamma rays in the decays $\pi^0 \rightarrow 2\gamma$. Since the π^0 lifetime is only about 10^{-16} sec, the π^0's never went more than about 3×10^{-6} cm before decaying. Consequently, in the present experiment, the origin of all gamma rays could be taken as the K^+ decay point. Each K^+ decay thus was tagged by the number of electron pairs associated with it.

In addition to the electron-pair tag given to each K^+ decay, decays were classified according to the nature of their charged secondaries. It was found that the best way to tag a charged secondary from a K^+ decay was to call it an e^+ or a non-e^+. An e^+ secondary was usually recognized by its large rate of energy loss due to multiple scattering and bremsstrahlung. It will be shown later that 97% of the e^+ secondaries from K_{e3} decays were recognized this way. The other 3% were either of sufficiently high energy or else were formed sufficiently close to the edge of the chamber that they left the chamber without exhibiting the characteristic e^+ behavior.

The K^+ decays which had non-e^+ secondaries were separated further according to whether the secondaries stopped, underwent a nuclear interaction, or simply left the chamber. However, the information gained from this additional separation did not play an important role in the determination of the K^+ branching ratios because if it had been used, the effects of scanning biases would have been increased, and also, additional Monte Carlo calculations would have been necessary. It should be pointed out that no reliable separation could be made between stopping π^+ and μ^+ secondaries on the basis of π^+-μ^+ decays because the μ^+ secondary from a π^+ decay at rest has a range of only 1.3 mm in xenon.

Thus in the initial scanning phase of the experiment, each K^+ decay was described by the number of electron pairs from gamma rays (0 to 4), and by the nature of the charged secondary (e^+ or non-e^+). All disagreements between the two independent scanners were resolved by rescanning with the help of a physicist.

Measuring

To gain further information about the K^+ decays obtained in the initial scan, all decays were measured, except those recognized in scanning as τ^+ decays by their three charged secondaries and those classified as zero-gamma non-e^+ in which the secondary left the chamber without undergoing a nuclear interaction. The measured events amounted to about 36% of all the events found in scanning.

The measuring was done by means of projectors which could project any one of the four stereoscopic views onto a large screen. The three best views were selected and the projected coordinates of the important points of a decay on the screen were punched onto IBM cards by means of electronic digitizers. These cards comprised the input to a computer program called KSORT3 (the third and final version of a "kaon sorting" program) which converted the projected coordinates into real space coordinates. Two stereo views are sufficient to determine the space coordinates of any

[5] J. L. Brown, H. C. Bryant, R. A. Burnstein, D. A. Glaser, R. Hartung, J. A. Kadyk, J. D. van Putten, D. Sinclair, G. H. Trilling, and J. C. Vander Velde, Nuovo Cimento **19**, 1155 (1961).

given point in the chamber, but three views were used, the third view giving an indication of the measurement errors and the care with which the measurement was made. If a measurement failed to meet specified requirements with regard to measurement quality, it was repeated.

The measuring program KSORT3 contains a kinematical test that deserves special mention because of the important role it played in this experiment. It is called the $K_{\pi 2}$ kinematical test and was applied to those K^+ decays which had exactly two gamma rays convert in the chamber, provided each of the two gamma rays convert at a distance greater than 3 mm from the K^+ decay point. Decays which had one or more gamma rays convert less than 3 mm from the decay point were not subjected to the $K_{\pi 2}$ kinematical test because the gamma-ray direction cosines could not be determined accurately.

The decay configuration is completely specified by the direction cosines of each of the two gamma rays and the charged secondary, relative to an arbitrary coordinate system having the K^+ decay point as the origin. Since the $K_{\pi 2}$ decay mode is the only two-body decay mode having a π^0, the idea of the $K_{\pi 2}$ kinematical test is to find out if the observed decay configuration could belong to a two-body decay. The first step is to check for coplanarity because in a $K_{\pi 2}$ decay with the K^+ at rest, the π^+ and the π^0 must go in opposite directions to conserve momentum. Consequently, the initial π^+ direction must lie in the plane defined by the directions of the two gamma rays from the π^0. For the decay being tested, the angle between the direction of the charged secondary and the plane of the two gammas is computed. This angle is called DEVCOP (deviation from coplanarity) and is expressed in degrees. The second step is to take the measured angle between the two-gamma-ray directions and compute the direction in which the π^0 went, using the constraints imposed by conservation of momentum and energy in the decay $\pi^0 \rightarrow 2\gamma$, and assuming that the π^0 came from a $K_{\pi 2}$ decay. The angle between the computed π^+ direction, which is opposite to that of the π^0, and the measured direction taken by the charged secondary is called ERFIT (fitting error) and is expressed in degrees. The two angles, DEVCOP and ERFIT, were combined to form a single quantity called KINFIT (kinematics fit). KINFIT is arbitrarily defined to be the square root of the sum of the squares of DEVCOP and ERFIT.

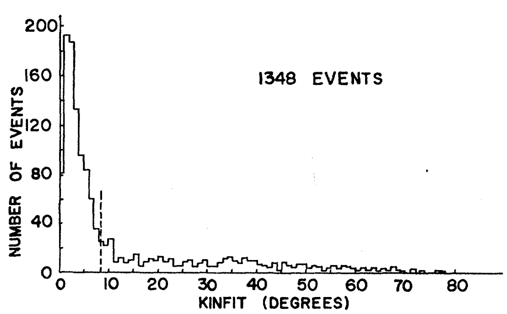

FIG. 1. The distribution of KINFIT for all the two-gamma events used in the present experiment. The arbitrary borderline between events that pass and events that fail $K_{\pi 2}$ kinematics is represented by the dashed line at 8.5°.

TABLE II. Probability of passing $K_{\pi 2}$ kinematics given that precisely two gamma rays convert in the chamber.

Decay mode	Probability
$K_{\pi 2}$	0.925±0.008
$K_{\mu 3}$	0.094±0.002
K_{e3}	0.075±0.002
τ'	0.013±0.001

Figure 1 is a histogram showing the variation of KINFIT for the two-gamma events used in the present experiment. An arbitrary cutoff of 8.5° for KINFIT was selected, meaning that all two-gamma events having KINFIT$\leq$8.5° were said to pass $K_{\pi 2}$ kinematics, and all the two-gamma events having KINFIT$>$8.5° were said to fail $K_{\pi 2}$ kinematics. Of all the two-gamma events used in the current experiment 82.8% passed $K_{\pi 2}$ kinematics. Table II gives the probability that a two-gamma event of the indicated decay mode will pass $K_{\pi 2}$ kinematics. These probabilities were computed using Monte Carlo decays as will be discussed later in more detail. It should perhaps be mentioned here that a two-gamma $K_{\pi 2}$ does not have a probability of 1.0 of passing $K_{\pi 2}$ kinematics because of measurement errors, multiple scattering of the charged secondary, and the inclusion of decays in flight which were thought to be at rest. The reason that a small fraction of $K_{\mu 3}$, K_{e3}, and τ' decays passed $K_{\pi 2}$ kinematics is that the decay configurations such that KINFIT is less than 8.5° are not forbidden.

The results of the $K_{\pi 2}$ kinematical test depend on the measurement errors. To subject an event with two gamma rays converting in the chamber to the $K_{\pi 2}$ kinematical test, the coordinates relative to the K^+ decay point of the following three points are needed; the first measured point on the charged secondary and the conversion points of each of the two gamma rays. To determine how measurement error affects these relative coordinates, a study of multiple measured events was made, yielding the following values for the standard deviation from their true values of the measured coordinates: 0.029 cm for the X and Y, and 0.062 cm for the Z coordinates. The Z coordinate is subject to more error because the Z direction is optically perpendicular to the plane of the four cameras.

Final Scan

After the initial scanning and measuring phases of the experiment were completed, two physicists independently rescanned and checked approximately 20%

of the decays used in the present experiment, including all the difficult and ambiguous decays. The purpose was to evaluate and determine the best way to reduce the scanning and measuring biases. The events which the physicists found to be scanned or measured incorrectly were corrected.

III. SELECTION OF K^+ DECAYS

The 21 000 events found by the initial scan and corrected by the final scan, of which about 36% were measured, make up the initial decay sample. The following discussion deals with the selection of about half of these events to form the final decay sample from which the branching ratios were computed.

The maximum number of decays per picture was reduced from six to four to further reduce gamma-ray and secondary ambiguities.

The final decay sample includes only decays that occurred at least 5 cm from every wall of the chamber. This increases the electron recognition efficiency and the gamma-ray conversion probability. It also reduces the π^+ contamination.

Decays classified as zero-gamma e^+ are not included in the final decay sample because such events are difficult to distinguish from π^+'s that entered the chamber with the beam and subsequently decayed. The absence of zero-gamma e^+ decays in the final decay sample is not serious because the expected number of zero-gamma e^+ decays was calculated by the Monte Carlo method, as will be discussed later.

Analysis of the dip angle for difficult and ambiguous secondaries revealed that about one-third of them have a Z-direction cosine greater than 0.9 in absolute value. Hence, all decays for which the charged secondary has a Z-direction cosine greater than 0.9 in absolute value were discarded. The criterion, of course, does not apply to τ^+ decays since these decays have three charged secondaries. This raises the statistical errors in the experiment by about 6%, but it eliminates a large fraction of the ambiguous events, raises the electron recognition efficiency, and eliminates some of the systematic errors in distinguishing between e^+ and non-e^+ secondaries. The number of ambiguous secondaries remaining in the final decay sample is about 37, and were divided equally between the e^+ and non-e^+ categories.

This completes the discussion of selection criteria that eliminate whole events. The remaining selection criteria pertain to the electron pairs from gamma rays.

Electron pairs from gamma rays are required to have at least 2 cm of potential path in the chamber to allow sufficient room for positive identification. Electron pairs not satisfying this condition were discarded. Electron pairs having a Z-direction cosine greater than 0.8 in absolute value were also discarded. This eliminated about one-third of the ambiguous electron pairs and meant that in the range -0.8 to $+0.8$ for gamma-ray Z-direction cosines, the scanning efficiency, while not 100%, is at least isotropic. This fact alone turned out to be very helpful in the evaluation of the over-all gamma-ray scanning bias.

IV. BIAS CORRECTION

Monte Carlo Calculations

To aid in making corrections to the data, a large number of Monte Carlo $K_{\pi 2}$, $K_{\mu 3}$, K_{e3}, and τ' decays (all decays involving gamma rays) were generated using an IBM-7090 digital computer, their distribution throughout the chamber being based on the experimentally measured distribution of K^+-decay points. In order to simulate the real decays as closely as possible, the Monte Carlo decays have the following properties built into them:

(1) The distribution of track segments (distance from the decay point to the first measured point on the charged secondary) is based on the experimentally measured track segment distribution.

(2) All points of the Monte Carlo decays were given measurement errors in accordance with the experimentally determined measurement errors.

(3) The charged secondaries of the Monte Carlo decays were given a directional error which is based on the known multiple scattering of charged particles in xenon.[6]

(4) The decays, $\pi^0 \to 2\gamma$, are isotropic in the π^0 center-of-mass system.

(5) The conversion distances of gamma rays in xenon were assigned according to the experimentally determined distribution which was obtained by the study of gamma rays of known energy from $K_{\pi 2}$ decays.

(6) The Monte Carlo programs contain as an input parameter, the K^+ momentum, allowing decays in flight to be simulated.

(7) In generating $K_{\mu 3}$ and K_{e3} decays, the π^0 momentum and the angle between the π^0 and the neutrino are obtained from the vector-axial vector coupling distribution, assuming variable form factors with the latest experimental energy dependence.[7] For τ' decays, phase space is assumed to govern the decay configuration.

The Monte Carlo decays and the gamma rays associated with them were subjected to exactly the same selection criteria used to select the actual experimental decays. Those Monte Carlo decays that had exactly two gamma rays convert in the chamber were put to the $K_{\pi 2}$ kinematical test discussed earlier. This is how the probabilities of two-gamma $K_{\pi 2}$, $K_{\mu 3}$, K_{e3}, and τ' decays passing $K_{\pi 2}$ kinematics were obtained, the results being given in Table II.

[6] B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952), p. 68.
[7] G. L. Jensen, F. S. Shaklee, B. P. Roe, and D. Sinclair, succeeding paper, Phys. Rev. **136**, B1431 (1964).

Low-Energy Gamma Rays

The lower energy gamma ray in a $K_{\pi 2}$ has an energy spectrum ranging from 20 to 120 MeV. To investigate a possible scanning bias against low-energy electron pairs, the energy spectrum of the lower energy electron pair from the Monte Carlo $K_{\pi 2}$ decays was compared with the corresponding spectrum from the measured two-gamma $K_{\pi 2}$ decays. Below 70 MeV, the experimental spectrum exhibited a defficiency in low-energy electron pairs which is attributed to a scanning bias against low-energy electron pairs. This scanning bias was evaluated as a function of energy and incorporated in all further Monte Carlo calculations.

Other Missed Gamma Rays

In addition to low-energy bias, the Monte Carlo decays were corrected for over-all gamma-ray scanning biases. It was estimated that aside from low-energy and dipped gammas, the physicists' scanning efficiency is $(98\pm1)\%$, while the professional scanners' efficiency is $(95\pm1)\%$. These estimates are based on the observed 1-gamma/2-gamma ratio for $K_{\pi 2}$ and $K_{\mu 3}$ decays.

K_{e3}-τ' Ambiguity

It was estimated that about 17% of the charged pions from τ' decays have momenta less than 34 MeV/c and hence ranges less than 1 mm in xenon. From observing π^+-μ^+-e^+ decays in which the μ^+ has a constant range of 1.3 mm, it is known that the efficiency for detecting the μ^+ was less than 50%. It is therefore important to know how many short pions from τ' decays were missed in scanning, causing the τ' decays to be called K_{e3}'s. The physicists carefully scanned 142 π^+'s from τ decays and found 10 that would be called e^+'s if it were not known that they came from τ decays and hence had to be π^+'s. The statistical fluctuation in these 10 events is about 3.1; therefore, it was estimated that 10 ± 3.1 out of 142 or $(7\pm2)\%$ of the 0-, 1-, and 2-gamma τ' decays had been called K_{e3} decays in scanning. The π^+-momentum spectrum for τ' decays[8] was compared with the π^+-momentum spectrum for τ decays[9] for low-pion momenta. The difference is certainly not enough to affect the above estimate of $(7\pm2)\%$.

K_{e3}-$K_{\mu 3}$ Ambiguity

From the $K_{\mu 3}$ muon spectrum, it was estimated that approximately 1% of the muons from $K_{\mu 3}$ decays have momenta less than 28 MeV/c and hence ranges less than 1 mm in xenon. Therefore, the fraction of $K_{\mu 3}$ decays that were called K_{e3} decays in scanning was estimated to be $(0.5\pm0.5)\%$.

[8] G. E. Kalmus, A. Kerman, W. M. Powell, R. Dowd, and W. F. Fry, Bull. Am. Phys. Soc. **9**, 34 (1964).

[9] S. McKenna, S. Natali, M. O'Connell, J. Tietge, and N. C. Varshnaya, Nuovo Cimento **10**, 763 (1958).

Decays in Flight

As will be seen later, the calculation of the $K_{\pi 2}$ and $K_{\mu 3}$ branching ratios is very sensitive to $P(K_{\pi 2})$, the probability that a two-gamma $K_{\pi 2}$ decay will pass $K_{\pi 2}$ kinematics. It was mentioned earlier that there are three reasons why a measured $K_{\pi 2}$ decay can fail $K_{\pi 2}$ kinematics: measurement errors, multiple scattering of the charged secondary, and the inclusion of decays in flight which were thought to be at rest. The effects of measurement errors and multiple scattering are built into the Monte Carlo programs, as explained earlier. The following discussion deals with the contribution that undetected decays in flight make to the failure rate of real $K_{\pi 2}$ decays subjected to the $K_{\pi 2}$ kinematical test.

A K^+ decay in xenon can usually be recognized in scanning as being in flight or at rest by examining the bubble density and multiple scattering of the last few centimeters of K^+ track. From the average range of the kaons in xenon, the average time for the kaons to come to rest was calculated, giving 8.92×10^{-10} sec. Using 1.22×10^{-8} sec as the K^+ lifetime, the probability that a K^+ will decay before coming to rest is

$$1-\exp(-8.92\times10^{-10}/1.22\times10^{-8})=0.0705.$$

Thus about 7% of the K^+ mesons entering the chamber decayed before coming to rest. All K^+ decays in flight that were detected in scanning were discarded. However, a K^+ decaying in flight with a short residual range (less than about 3 cm) cannot be distinguished in scanning from a decay at rest. In fact, in the final sample of 895 two-gamma non-e^+ events, 54 were called in flight in scanning, which amounts to $(6\pm1)\%$, where the error is due to the statistical fluctuation of 54 out of 895 events. The fraction of undetected decays in flight is therefore about $(1\pm1)\%$. This means that about $(1\pm1)\%$ of the final sample of K^+ decays used in the present experiment are in flight, even though all K^+ decays recognized in scanning as being in flight were discarded. The only place that these undetectable decays in flight affect the current experiment is in the contribution that they make to the chance of a real $K_{\pi 2}$ decay failing $K_{\pi 2}$ kinematics.

Let X be defined as the residual range cutoff, i.e., if the kaons decay with a residual range greater than X, they are detectable in scanning as decays in flight and discarded. If the residual range is less than X, the decays are not detectable as decays in flight and are kept. By assuming a value for X, one can calculate $P(K_{\pi 2})$, the probability that a measured $K_{\pi 2}$ decay will pass $K_{\pi 2}$ kinematics, by numerically evaluating the integral

$$P(K_{\pi 2})=\int_0^X P_1(y)P_2(y)dy,$$

where $P_1(y)dy$ is the probability that a kaon will decay with residual range between y and $y+dy$ in xenon, and $P_2(y)$ is the probability that a measured $K_{\pi 2}$ decay with

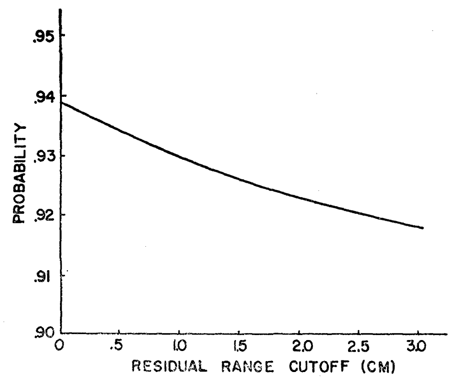

FIG. 2. Probability of a two-gamma $K_{\pi 2}$ passing $K_{\pi 2}$ kinematics as a function of the residual range cutoff.

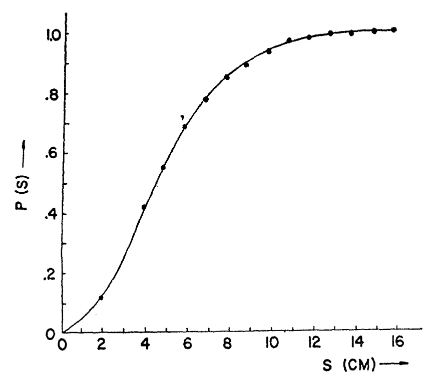

FIG. 3. $P(s)$, the probability that an electron secondary with potential path s will be recognized.

residual range y will pass $K_{\pi 2}$ kinematics. $P_1(y)$ was obtained from the known kinematics of a K^+ stopping in xenon. $P_2(y)$ was obtained by generating Monte Carlo $K_{\pi 2}$ decays in flight with a known K^+ momentum and subjecting them to the $K_{\pi 2}$ kinematical test. The variation of $P(K_{\pi 2})$ as a function of X is shown in Fig. 2. Using the kinematics of a K^+ stopping in xenon, one can show that the fraction of undetected decays in flight, $(1\pm 1)\%$, corresponds to a residual range cutoff of about 1.5 ± 1.5 cm, which from Fig. 2 gives $P(K_{\pi 2}) = 0.925$. The upper limit of 3.0 cm for the residual range cutoff was verified by the physicists who scanned about 60 K^+ decays at rest, estimating the residual range cutoff for each. Combining the error due to the present analysis with the measurement errors obtained by simulation of real $K_{\pi 2}$'s by Monte Carlo $K_{\pi 2}$'s gives the final value

$$P(K_{\pi 2}) = 0.925\pm 0.008.$$

Electron Recognition Efficiency

The following discussion deals with the corrections made for unrecognized K_{e3} decays. The probability of recognizing an electron secondary from a K_{e3} decay in xenon depends on the electron energy and the potential path available to the electron. Each electron secondary in the final sample of 436 one- and two-gamma recognized K_{e3}'s was scanned by a physicist who measured the minimum distance in which the electron could be recognized. This projected distance was converted to a distance in real space which will hereafter be called the electron recognition distance. Zero-gamma K_{e3}'s were excluded because of the possibility of confusing them with beam pions. The reader is reminded that the final sample excludes all decays occurring less than 5 cm from the chamber boundaries. It also excludes all dipped electron secondaries, i.e., those having Z-direction cosines greater than 0.9 in magnitude.

If r is the electron recognition distance, the number $n(x,y)$ is defined as the number of electrons having at least y cm of potential path and having $x\leq r\leq y$. Using different values for x and y, the numbers $n(x,y)$, obtainable directly from the data, were used to calculate $P(S)$, the probability of recognizing an electron within S cm from the decay point assuming only that the probability is 1 of recognizing a 20-cm-long electron track in xenon. $P(S)$ is shown in Fig. 3.

The electron recognition efficiency c_e was obtained by numerically evaluating the integral

$$c_e = \int_{S_{\min}}^{S_{\max}} P(S)P_1(S)dS = 0.974\pm 0.010,$$

where $P(S)$ is defined above, $P_1(S)dS$ is the probability that the electron from a one- or two-gamma $K_{e3}{}^+$ will have potential path between S and $S+dS$, and $S_{\max}$ and $S_{\min}$ are the maximum and minimum values of potential path available to the electrons ($S_{\min}=5$ cm). $P_1(S)dS$ was calculated by the $K_{e3}{}^+$ Monte Carlo program.

V. CALCULATION OF THE K^+ BRANCHING RATIOS

The fiducial volume is defined as that region in the chamber containing all points which are at least 5 cm from every wall of the chamber. As mentioned earlier, only those K^+ decays which occurred inside this fiducial volume were used in the present experiment. Since the τ decay mode was recognized in scanning by its three charged secondaries, no measurements of τ decays were made. Not all of the zero-gamma events were measured, and the zero-gamma K_{e3} decays that were found were discarded because of the possibility of confusing them with beam pions. Thus it was necessary to estimate the number of decays that occurred in the fiducial volume, using the experimental distribution of decay points. This was done, giving $10\,513\pm 141$ decays in the fiducial volume.

The τ Branching Ratio

From the total number of τ decays that occurred in the chamber, the number of τ decays that occurred in the fiducial volume was calculated, giving 540 ± 23. From the total number of decays in the fiducial volume, the τ branching ratio was calculated. The result was 0.051 ± 0.002.

The τ' Branching Ratio

The number of K^+ decays having three or four gamma rays, after all selection criteria were applied, is 108 ± 10. From the τ' Monte Carlo program, this number is $(57.0\pm1.8)\%$ of all the τ' decays. Hence the total number of τ' decays is 189 ± 19 and the τ' branching ratio is 0.018 ± 0.002.

The K_{e3} Branching Ratio

The number of one- and two-gamma decays with secondaries identified as electrons and satsifying all selection criteria is 436 ± 23. Since $(7\pm2)\%$ of the one- and two-gamma τ' decays and $(0.5\pm0.5)\%$ of the one- and two-gamma $K_{\mu3}$ decays were called K_{e3} decays because of their low-energy charged secondaries, the 436 ± 23 includes 6 ± 1 τ' decays and 1 ± 1 $K_{\mu3}$ decays. Hence the actual number of recognized K_{e3} decays is 429 ± 23.

According to the electron recognition efficiency, the number of recognized K_{e3}'s, 429 ± 23, is only $(97.4\pm1.0)\%$ of the actual number of K_{e3}'s. The e^+ secondaries from the remaining 2.6% left the chamber without being recognized. Hence the actual number of one- and two-gamma K_{e3}'s, both recognized and unrecognized, is 440 ± 24.

From the K_{e3} Monte Carlo program, the number of one- and two-gamma K_{e3}'s, 440 ± 24, is $(89.4\pm1.0)\%$ of all the K_{e3} decays. Hence the total number of K_{e3} decays, satisfying all selection criteria, is 493 ± 28, giving the K_{e3} branching ratio of 0.047 ± 0.003.

The $K_{\mu3}$ Branching Ratio

The total number of two-gamma events with secondaries identified as pions or muons is 940. Subtracting the number of two-gamma unrecognized K_{e3}'s (4) and the number of two-gamma τ' decays (50) leaves 886 ± 6 as the sum of the two-gamma $K_{\mu3}$ and $K_{\pi2}$ decays only. The error is due only to the errors in the subtracted events. Similarly, the sum of the two-gamma $K_{\mu3}$ and $K_{\pi2}$ modes which pass the $K_{\pi2}$ kinematical test is 734. An error of ±11 represents the expected statistical fluctuation of 734 out of 886 events.

From the $K_{\pi2}$ and $K_{\mu3}$ Monte Carlo programs, the probabilities of a two-gamma $K_{\pi2}$ and a two-gamma $K_{\mu3}$ passing the $K_{\pi2}$ kinematical test are 0.925 ± 0.008 and 0.094 ± 0.002, respectively. Let $B(K_{\pi2})=$ the $K_{\pi2}$ branching ratio, $B(K_{\mu3})=$ the $K_{\mu3}$ branching ratio, and $R=B(K_{\pi2})/B(K_{\mu3})$. Then $[R/(R+1)](0.925\pm0.008)$ is the probability that if K_{e3} and τ' decays are excluded, a two-gamma event that passes $K_{\pi2}$ kinematics is a $K_{\pi2}$. $[1/(R+1)](0.094\pm0.002)$ is the corresponding probability for $K_{\mu3}$ decays. Therefore,

$$\frac{R}{R+1}(0.925\pm0.008)+\frac{1}{R+1}(0.094\pm0.002)=(734\pm11)/(886\pm6).$$

Solving for R gives $R=7.6\pm1.3$.

The number of one-gamma decays with non-e^+ secondaries is 1518 ± 37. Subtracting the number of one-gamma unrecognized e^+ events, 7 ± 3, and the number of one-gamma τ' events, 24 ± 3, leaves 1487 ± 37 which is the sum of the one-gamma $K_{\pi2}$ and $K_{\mu3}$ events and is $(14.1\pm0.4)\%$ of the total number of events. From the $K_{\pi2}$ and $K_{\mu3}$ Monte Carlo programs, the probabilities that a $K_{\pi2}$ and a $K_{\mu3}$ will have only one gamma convert and be observed in the chamber are 0.551 ± 0.004 and 0.552 ± 0.005, respectively. Hence

$$B(K_{\pi2})(0.551\pm0.004)+B(K_{\mu3})(0.552\pm0.005)=0.141\pm0.004.$$

Knowing $R=B(K_{\pi2})/B(K_{\mu3})=7.6\pm1.3$, one can solve for $B(K_{\mu3})$ and obtain $B(K_{\mu3})=0.030\pm0.005$.

The $K_{\pi2}$ Branching Ratio

The $K_{\pi2}$ branching ratio can be obtained from $B(K_{\mu3})$ and the ratio $R=B(K_{\pi2})/B(K_{\mu3})$. However, it is more accurate to first calculate $S=B(K_{\pi2})+B(K_{\mu3})$ and obtain $B(K_{\pi2})$ from $B(K_{\pi2})=S-B(K_{\mu3})$.

The number of one- and two-gamma $K_{\pi2}$ and $K_{\mu3}$ events is 2372 ± 45. Adding the number of zero-gamma $K_{\pi2}$ and $K_{\mu3}$ events, obtained from the Monte Carlo programs, gives a total of 2666 ± 78 events which are pure $K_{\pi2}$ and $K_{\mu3}$ events. Hence

$$S=B(K_{\pi2})+B(K_{\mu3})=0.254\pm0.007,$$

from which

$$B(K_{\pi2})=0.224\pm0.008.$$

The $K_{\mu2}$ Branching Ratio

The most accurate way to calculate the $K_{\mu2}$ branching ratio is to subtract all of the previously obtained branching ratios from 1, giving 0.630 ± 0.008.

Sensitivity of the $K_{\pi2}/K_{\mu3}$ Branching Ratio

As mentioned earlier, R, the $K_{\pi2}/K_{\mu3}$ branching ratio, is very sensitive to $P(K_{\pi2})$, the probability that a two-gamma $K_{\pi2}$ will pass $K_{\pi2}$ kinematics. In fact, a major disagreement between the present experiment and A is in the value of R. Therefore, a brief discussion of the reason for the sensitivity and the care which was taken to reduce its effects will follow.

The equation used to calculate R is

$$\frac{R}{R+1}P(K_{\pi2})+\frac{1}{R+1}P(K_{\mu3})=F,$$

where F is the fraction of the sample of pure $K_{\pi2}$ and $K_{\mu3}$ decays that passed the $K_{\pi2}$ kinematical test. Solving for R gives

$$R=\frac{F-P(K_{\mu3})}{P(K_{\pi2})-F}.$$

The sensitivity comes from the denominator because the values of $P(K_{\pi 2})$ and F are close together. In fact,

$$P(K_{\pi 2})=0.925\pm 0.008$$

and

$$F=0.829\pm 0.014.$$

Subtracting

$$P(K_{\pi 2})-F=0.096\pm 0.016.$$

The major contribution to the error in $P(K_{\pi 2})-F$ comes from the statistical error in F, which is about 1.8%. However, the error in $P(K_{\pi 2})-F$ is 17%, which makes the error in the $K_{\mu 3}$ branching ratio alone about 15% and the error in the $K_{\mu 3}/K_{e3}$ branching ratio about 16%. Furthermore, a systematic error of 1% in determining $P(K_{\pi 2})$ reflects itself as a 10% systematic error in the $K_{\mu 3}$ branching ratio. For this reason, a large amount of effort has been made to reduce any systematic error in $P(K_{\pi 2})$ as much as possible. It is believed that a more accurate calculation of $P(K_{\pi 2})$ has been made in the present experiment than in A.

VI. RESULTS AND CONCLUSIONS

The results of the present experiment have been summarized in Table I. The number which are of particular interest to present day theorists are the $K_{\mu 2}$, $K_{\mu 3}/K_{e3}$, and τ'/τ branching ratios.

The $K_{\mu 2}$ Branching Ratio

Sugawara[10] has postulated a universality in the following decays:

$$\pi^{\pm}\to e^{\pm}+\nu \quad K^{\pm}\to e^{\pm}+\nu$$
$$\to \mu^{\pm}+\nu \qquad \to \mu^{\pm}+\nu(K_{\mu 2}),$$

which predicts a $K_{\mu 2}$ branching ratio of 0.677 ± 0.011. This predicted $K_{\mu 2}$ branching ratio was calculated by the present authors and does not include some theoretical corrections, the most important of which is the electromagnetic correction. The error represents the latest errors in the lifetimes and masses of the particles involved. The present experimental value of the $K_{\mu 2}$ branching ratio is 0.630 ± 0.008.

The $K_{\mu 3}/K_{e3}$ Branching Ratio

The three-body leptonic decay modes of the K^+ meson, $K^+\to \mu^+ +\pi^0+\nu$ $(K_{\mu 3})$ and $K^+\to e^+ +\pi^0+\nu$ (K_{e3}), are presently assumed to be governed by a decay interaction that is the product of a leptonic current, restricted just to vector and axial vector couplings, and a strangeness nonconserving current due to strongly interacting particles. In the approximation that the lepton and neutrino arise from a local interaction, the amplitude for the leptonic decay of the K^+ meson is[11]

$$[\tfrac{1}{2}(P_K+P_\pi)f_+ +\tfrac{1}{2}(P_K-P_\pi)f_-]_\lambda \bar{u}(P_\nu)\gamma_\lambda(1+\gamma_5)v(P_L),$$

[10] M. Sugawara, Purdue University (to be published).
[11] N. Brene, L. Egardt, and B. Qvist, Nucl. Phys. **22**, 553 (1961).

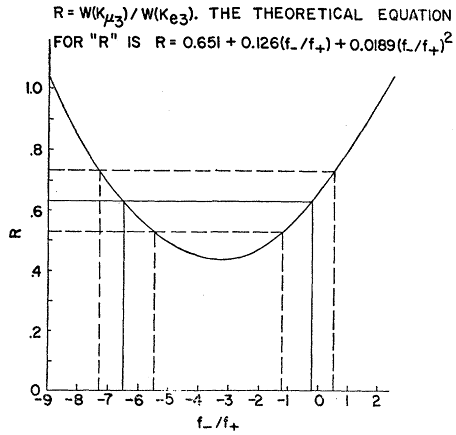

FIG. 4. Experimental solution of the theoretical equation for R. Horizontal lines represent the experimental value, $R=0.63\pm 0.10$.

where P_π and P_K are the pion and kaon four-momenta, u and v are the Dirac spinors for the leptons, and f_+ and f_- can be scalar functions (called form factors) of $q^2=(P_K-P_\pi)^2$, the square of the invariant four-momentum transfer. Since the decays occurred at rest in the laboratory, f_+ and f_- can be considered functions only of the pion momentum. If the weak interaction is time-reversal invariant, f_-/f_+ must be real. The ratio f_-/f_+ determines the ratio of the $K_{\mu 3}{}^+$ and $K_{e3}{}^+$ branching ratios in accordance with[12]

$$R=B(K_{\mu 3}{}^+)/B(K_{e3}{}^+)=0.651+0.126(f_-/f_+) +0.0189(f_-/f_+)^2,$$

assuming that f_+ and f_- vary slowly with q^2, an assumption that has experimental verification.[7]

The present experimental value (which, incidentally, is lower than all the previous experimental values) gives

$$R=0.63\pm 0.10,$$

which gives two solutions for f_-/f_+. One solution is consistent with $f_-=0$ and is

$$f_-/f_+=-0.17_{-0.99}^{+0.75}.$$

The other solution is not consistent with $f_-=0$ and is

$$f_-/f_+=-6.49_{-0.75}^{+0.99}.$$

The two solutions are depicted graphically in Fig. 4.

The present experiment agrees with the predictions[13] of $R=0.69$ on the basis of an S-wave K-π resonance at 880 MeV, and $R=0.64$ on the basis of a P-wave K-π resonance at 880 MeV.

The τ'/τ Branching Ratio

The three-pion decays of the K^+ meson afford an opportunity to check the $\Delta T=\frac{1}{2}$ rule. Using the fact that the K spin is zero, the generalized Bose statistics

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[13] H. Chew, Phys. Rev. Letters **8**, 297 (1962).

and the $\Delta T=\frac{1}{2}$ rule say that the final three-pion isotopic spin states must be symmetric, giving amplitudes of 1 and 2 for τ' and τ decays, respectively. Thus the τ'/τ branching ratio is $\frac{1}{4}$. The correction for the difference in phase space in the τ' and τ modes contributes a factor[14] of 1.298, making the τ' branching ratio equal to

$$B(\tau')/B(\tau)=\tfrac{1}{4}(1.298)=0.325\ .$$

The experimental value is 0.350 ± 0.039 which is consistent with the $\Delta T=\frac{1}{2}$ rule.

$K_{\mu3}+K_{e3}$ Branching Ratio

A test of the leptonic $\Delta T=\frac{1}{2}$ rule comes from comparing $K_2{}^0$ and K^+ decays.[15,16] The $\Delta T=\frac{1}{2}$ rule predicts

$$\Gamma(K_2{}^0\to\pi^\pm+e^\mp+\nu)+\Gamma(K_2{}^0\to\pi^\pm+\mu^\mp+\nu)$$
$$=2[\Gamma(K^+\to\pi^0+e^++\nu)+\Gamma(K^+\to\pi^0+\mu^++\nu)].$$

Luers *et al.*[15] quote a weighted average of world data up to 1964 for the left side of the equation of $9.9\pm2.0\times10^6$/ sec, while Mann[17] quotes a later experimental value of $13.3\pm2.5\times10^6$/sec. A weighted average of these two yields:

$$\Gamma(K_2{}^0\to\pi^\pm+e^\mp+\nu)+\Gamma(K_2{}^0\to\pi^\pm+\mu^\mp+\nu)$$
$$=(11.2\pm1.6)\times10^6/\text{sec}.$$

For the right side of the equation we obtain

$$2[\Gamma(K^+\to\pi^0+e^++\nu)+\Gamma(K^+\to\pi^0+\mu^++\nu)$$
$$=(12.6\pm1.0)\times10^6/\text{sec}$$

using the K^+ lifetime quoted by Barkas and Rosenfeld.[18]

The difference between these two is $(1.4\pm1.9)\times10^6$/ sec, consistent with $\Delta T=\frac{1}{2}$.

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Study of the Three-Body Leptonic Decay Modes of the K^+ Meson*

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$K_{e3}{}^+$ and $K_{\mu3}{}^+$ decay spectra are studied. The results strongly favor vector coupling even if the form factors are allowed arbitrary energy dependence. The relative amplitudes of scalar and tensor couplings are less than 0.3 with the most probable value around 0. The muon and electron couplings are found to be the same within the 14% error quoted. All energy dependences of the form factors are found to be small. If $f_V=A(1+\lambda q^2/M_\pi{}^2)$ and $g_V=B$, where $q^2=(p^K-p^\pi)^2$, then $\lambda=-0.020\pm0.027$, $B/A=-0.54\pm0.35$.

I. INTRODUCTION

In this paper we examine in detail the three-body leptonic decay modes of the K^+meson

$$(1)\quad K^+\to e^++\nu+\pi^0\quad(K_{e3}{}^+)\,,$$
$$(2)\quad K^+\to\mu^++\nu+\pi^0\quad(K_{\mu3}{}^+)\,.$$

In the preceding paper[1] (hereafter called I) we found the branching ratios for the above modes to be 4.7%, and 3.0%, respectively.

Two general but related types of information are obtained in this experiment. The first concerns the nature of the decay interaction, in particular the type of coupling responsible for the decay. Previous experiments[2–4] indicate that the interaction is consistent with pure vector coupling and that pure scalar and pure tensor couplings are much less likely to be present than vector. Further confirmation on this point is available in the current investigation, and limits on the amount of scalar and tensor mixing with the dominant vector coupling are also obtained. The second type of information relates to the energy dependence of the form factors needed to describe the interaction and to the coupling constants involved.

We wish to emphasize that this experiment has been done on a different sample of film from that used for our previous leptonic decay spectrum work,[2] and has

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